

# Anomaly-free flavor models for Nambu-Goldstone bosons and the 3.5 keV X-ray line signal

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## Abstract

We pursue a possibility that a pseudo-Nambu-Goldstone boson is lurking around or below the intermediate scale. To this end we consider an anomaly-free global flavor symmetry, and construct models where the pseudo-Nambu-Goldstone boson is coupled preferentially to leptons. The experimental and astrophysical bounds derived from couplings to photons and nucleons are significantly relaxed. If sufficiently light, the pseudo-Nambu-Goldstone boson contributes to dark matter, and interestingly, it generally decays into photons through couplings arising from threshold corrections. We show that the recent hint for the X-ray line at about 3.5 keV can be explained by the decay of such pseudo-Nambu-Goldstone boson of mass about 7 keV with the decay constant of order  $10^{10}$  GeV, if the electron is charged under the flavor symmetry.

# 1 Introduction

Symmetry plays an important role in physics. Sometimes it is spontaneously broken in the low energy, and as a remnant, there appears a massless Nambu-Goldstone boson. If the symmetry is a local one, it is absorbed by the corresponding gauge boson. On the other hand, if the symmetry is a global and approximate one, there remains a light pseudo-Nambu-Goldstone boson (PNGB), which has been a subject of considerable interest. PNGBs, if exist, will provide us with invaluable information on the high energy physics.

Various types of global symmetries and the associated PNGBs have been considered so far. One example is the QCD axion, which arises in association with the spontaneous breakdown of the Peccei-Quinn symmetry [1, 2]. Importantly, the QCD axion is coupled to gluons and photons through anomalies, as well as to the quarks and the leptons at tree or one-loop level. The interactions are suppressed by the decay constant, which parametrizes the symmetry breaking scale. In other extensions of the standard model (SM), there arise PNGBs with similar properties, the so called axion-like particles, and especially those with couplings to photons have been studied extensively from both theoretical and experimental aspects [3].

The couplings of the QCD axion and the axion-like particles to photons, nucleons, and electrons are tightly constrained by cosmology, astrophysics and the ground-based experiments [3, 4, 5]. In particular, the astrophysical constraints are extremely tight, pushing the scale of new physics to an intermediate scale or above. Still, there may be other kind of PNGBs with different properties at a scale around or even below the intermediate scale, without any conflict with those constraints.

In this paper we pursue a possibility that a PNGB associated with new physics is lurking around or below the intermediate scale. For this, we need to evade tight astrophysical bounds on the PNGBs. One way is to consider PNGBs, which are not directly coupled to the SM sector, but mainly coupled to a hidden sector [6]. Instead, we want to consider here the case in which some of the SM particles are charged under a global flavor symmetry. The maximal possible flavor symmetry for the SM particles with three right-handed neutrinos is  $U(3)^6$ . We consider an anomaly-free global  $U(1)_F$  flavor symmetry, which is a subgroup of the maximal flavor symmetry. In particular, a leptophilic PNGB model is

simple and phenomenologically interesting, and we will construct concrete models along this lines. Such leptophilic PNGBs without anomalous couplings to photons evade various experimental and astrophysical bounds coming from couplings with nucleons and photons. We will mainly focus on very light PNGBs with mass lighter than the twice the electron mass.<sup>1</sup>

There is an interesting point of the PNGB associated with an anomaly-free global symmetry. Although suppressed, such PNGB is necessarily coupled to photons through threshold corrections. In particular, the decay into two photons can be the main decay mode if the PNGB of mass is less than twice the electron mass. If such light PNGB constitutes dark matter, it mainly decays into two photons, producing a narrow X-ray line. This can explain the recent hint for the X-ray line at about 3.5 keV [8, 9] for the PNGB mass of about 7 keV. As we shall see, the required decay constant is  $f_a = \mathcal{O}(10^{10})$  GeV if the electron is charged under the symmetry, whereas it is  $f_a = \mathcal{O}(10^5)$  GeV if the electron is neutral under the symmetry. This should be contrasted to the fact that the observed X-ray flux can also be explained by the string axion with a decay constant of order  $10^{14-15}$  GeV as first pointed out in Ref. [10].<sup>2</sup>

The rest of the paper is organized as follows. In Sec. 2 we discuss the coupling of the PNGB to photons through threshold corrections, and its implications for the 3.5 keV X-ray line. We discuss production of PNGB dark matter in Sec. 3. In Sec. 4, we will build concrete models for leptophilic PNGBs. The last section is devoted for discussion and conclusions.

## 2 Couplings of PNGBs to photons

Let us consider a global  $U(1)_F$  flavor symmetry under which only leptons are charged. Most important, we assume that the global  $U(1)_F$  symmetry is anomaly free so that the PNGB coupling to photons is suppressed, evading various observational constraints. The coupling to photons is nevertheless induced by threshold corrections, which we will study

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<sup>1</sup> Experimental bounds on PNGBs with mass heavier than  $\mathcal{O}(1)$  MeV including leptophilic ones were studied in Ref. [7].

<sup>2</sup> The X-ray line produced by light modulus decay was studied many years ago by Kawasaki and one of the present authors (TTY) in Ref. [11] (see also Refs. [12, 13]). Recently there appeared various possibilities to explain the 3.5 keV X-ray line [14, 10, 15, 16, 17, 18].

in this section.

Let us study the interactions of the PNGB with leptons in the low energy. Later we will construct concrete flavor models. The relevant low-energy interactions are given by

$$-\mathcal{L} = m_e \bar{e}_R e_L e^{iq_e \frac{a}{f_a}} + m_\mu \bar{\mu}_R \mu_L e^{iq_\mu \frac{a}{f_a}} + m_\tau \bar{\tau}_R \tau_L e^{iq_\tau \frac{a}{f_a}} + \text{h.c.} \quad (1)$$

where  $a$  is the PNGB associated with the flavor symmetry,  $f_a$  the decay constant, and  $q_e$ ,  $q_\mu$ , and  $q_\tau$  the coupling constants for electron, muon and tau leptons, respectively. We exclude the case of  $q_e = q_\mu = q_\tau = 0$  in the following analysis.

We are interested in the case where the PNGB mass is much lighter than twice the electron mass. Integrating out electron, muon, and tau leptons, therefore, we obtain the effective interaction,

$$\begin{aligned} \mathcal{L}_{\text{eff}} \simeq & -(q_e + q_\mu + q_\tau) \frac{\alpha_{em}}{4\pi f_a} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ & + \frac{\alpha_{em}}{48\pi f_a} \left( \frac{q_e}{m_e^2} + \frac{q_\mu}{m_\mu^2} + \frac{q_\tau}{m_\tau^2} \right) \left( (\partial^2 a) F_{\mu\nu} \tilde{F}^{\mu\nu} + 2a F_{\mu\nu} \partial^2 \tilde{F}^{\mu\nu} \right), \end{aligned} \quad (2)$$

where the first line corresponds to the anomaly term, and the second line arises from the threshold corrections. We require  $q_e + q_\mu + q_\tau = 0$  to ensure that the flavor symmetry is anomaly-free. Then the first term in Eq. (2) vanishes, and we are left with the finite threshold corrections. Therefore the PNGB coupling to photons is significantly suppressed for anomaly-free symmetry. As long as we are interested in the decay or production of the on-shell PNGB and photons, we can use their equations of motion. Then the effective interaction for the PNGB to photons becomes

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_{em} m_a^2}{48\pi f_a} \left( \frac{q_e}{m_e^2} + \frac{q_\mu}{m_\mu^2} + \frac{q_\tau}{m_\tau^2} \right) a F_{\mu\nu} \tilde{F}^{\mu\nu} \quad (3)$$

for the on-shell PNGB and photons and  $m_a^2 \ll m_e^2$ . The PNGB coupling to photons is dominated by the first term if  $q_e \neq 0$ ; otherwise it is dominated by the second term. Note that both  $q_e$  and  $q_\mu$  cannot vanish simultaneously to satisfy the anomaly-free condition.

The decay rate of the PNGB into two photons is approximately given by

$$\Gamma_{a \rightarrow \gamma\gamma} \simeq \frac{\alpha_{em}^2 m_a^7}{9216\pi^3 f_a^2} \times \begin{cases} q_e^2/m_e^4 & \text{for } q_e \neq 0, \\ q_\mu^2/m_\mu^4 & \text{for } q_e = 0 \end{cases} \quad (4)$$

where we have approximated  $m_e^2 \ll m_\mu^2 \ll m_\tau^2$  and assumed that there is no large hierarchy among the  $U(1)_F$  charges. Assuming that the PNGB decays mainly into photons via the above interaction, we can estimate the lifetime as

$$\tau_{a \rightarrow \gamma\gamma} \simeq \begin{cases} 2.9 \times 10^{28} q_e^{-2} \left( \frac{m_a}{7\text{keV}} \right)^{-7} \left( \frac{f_a}{10^{10} \text{GeV}} \right)^2 \text{sec.} & \text{for } q_e \neq 0 \\ 2.1 \times 10^{28} q_\mu^{-2} \left( \frac{m_a}{7\text{keV}} \right)^{-7} \left( \frac{f_a}{2 \times 10^5 \text{GeV}} \right)^2 \text{sec.} & \text{for } q_e = 0 \end{cases} \quad (5)$$

Thus the PNGB is so long-lived that it can contribute to dark matter. We will show in the next section that, in fact, the right amount of PNGBs can be produced to explain the observed dark matter abundance.

The recent hint for the X-ray line at about 3.5 keV can be explained by dark matter with the following mass and lifetime [8, 9]:

$$m_{\text{DM}} \simeq 7.1 \text{ keV}, \quad (6)$$

$$\tau_{\text{DM}} \simeq 4 \times 10^{27} - 4 \times 10^{28} \text{ sec}, \quad (7)$$

if it decays into a pair of photons. Therefore, the 3.5 keV X-ray line can be explained by the decay of the PNGB dark matter with  $m_a \simeq 7 \text{ keV}$  and  $f_a/q_e = 4 \times 10^9 \text{ GeV} - 1 \times 10^{10} \text{ GeV}$  for  $q_e \neq 0$ , or  $f_a/q_\mu = 9 \times 10^4 \text{ GeV} - 3 \times 10^5 \text{ GeV}$  for  $q_e = 0$  and  $q_\mu \neq 0$ . Interestingly, a relatively small decay constant below the intermediate scale is needed because of the suppression factor for the threshold corrections. This should be contrasted to the fact that the observed X-ray flux can also be explained by the string axion with a decay constant of order  $10^{14-15} \text{ GeV}$  [10].

### 3 PNGB dark matter

A light PNGB contributes to dark matter, if it is sufficiently long-lived. In order to explain the observed dark matter density, the right amount of PNGBs need to be produced in the early Universe. There are two important production processes. One is non-thermal production by the initial misalignment mechanism, and the other is thermal production.<sup>3</sup> We will consider these production processes in turn.

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<sup>3</sup>The production of PNGB dark matter was recently studied in Ref. [19].

The PNGB number density to entropy ratio can be written as

$$Y_a \simeq 6 \times 10^{-5} \left( \frac{m_a}{7\text{keV}} \right)^{-1} \left( \frac{\Omega_a h^2}{0.12} \right), \quad (8)$$

where  $\Omega_a$  is the density parameter for the PNGB and  $h$  is the reduced Hubble constant. On the other hand, if the PNGB is in equilibrium, its abundance is given by

$$Y_a^{(\text{eq})} \simeq 2.6 \times 10^{-3} \left( \frac{g_*}{106.75} \right)^{-1}, \quad (9)$$

where  $g_*$  counts the relativistic degrees of freedom in thermal plasma. Therefore, if the PNGBs constitute the observed dark matter, they should not be in equilibrium, otherwise there must be late-time entropy dilution by a factor 40 for  $m_a \simeq 7\text{keV}$ .

Let us first consider the case of  $q_e \neq 0$ . In this case, the decay constant suggested by the observed X-ray line is  $f_a/q_e = 4 \times 10^9 \text{ GeV} - 1 \times 10^{10} \text{ GeV}$ . The thermal production process depends on the charge of  $\tau$ . If the PNGB is directly coupled to  $\tau$ , the main production process will be through scatterings between leptons and Higgs bosons such as  $\ell_3 H^* \rightarrow a \tau_R$ . The abundance is roughly estimated as follows

$$Y_a^{(\text{th})} \sim 6 \times 10^{-5} \left( \frac{g_*}{106.75} \right)^{-1} \left( \frac{T_R}{10^6 \text{ GeV}} \right) \left( \frac{f_a}{10^{10} \text{ GeV}} \right)^{-2}, \quad (10)$$

where  $T_R$  is the reheating temperature. Thus, the right amount of PNGBs are thermally produced for  $T_R \sim 10^6 \text{ GeV}$  and  $f_a \sim 10^{10} \text{ GeV}$ . Alternatively, if the PNGB is not directly coupled to  $\tau$ , the abundance is suppressed by  $\sim (m_\mu/m_\tau)^2$  and given by

$$Y_a^{(\text{th})} \sim 2 \times 10^{-5} \left( \frac{g_*}{106.75} \right)^{-1} \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{f_a}{10^{10} \text{ GeV}} \right)^{-2}. \quad (11)$$

In this case successful thermal leptogenesis may be possible [20], with a mild degeneracy among the right-handed neutrinos. Note that the thermally produced PNGBs of  $7\text{keV}$  mass behave as warm dark matter because of their non-negligible free streaming.

The PNGBs can also be produced by the initial misalignment mechanism. The PNGB starts to oscillate when the Hubble parameter becomes comparable to the mass  $m_a$ . In the radiation dominated Universe, this happens when  $T \sim 2 \times 10^6 \text{ GeV} (m_a/7\text{keV})^{1/2}$ . Therefore, for  $T_R \lesssim 10^6 \text{ GeV}$ , the oscillations starts before the reheating, and the PNGB abundance is given by

$$Y_a^{(\text{mis})} \sim 3 \times 10^{-7} \left( \frac{T_R}{10^6 \text{ GeV}} \right) \left( \frac{m_a}{7\text{keV}} \right)^{-1} \left( \frac{f_a}{10^{10} \text{ GeV}} \right)^2 \theta_*^2, \quad (12)$$

where  $\theta_* \equiv a_{\text{ini}}/f_a$  denotes the initial oscillation amplitude. If the  $U(1)_F$  symmetry is spontaneously broken after inflation, we should replace  $\theta_*$  with its averaged value,  $\sqrt{\langle \theta_*^2 \rangle} = \pi/\sqrt{3}$ .<sup>4</sup> For  $T_R \gtrsim 2 \times 10^6$  GeV, the abundance of PNGBs produced by the initial misalignment mechanism becomes independent of  $T_R$ . Therefore, the initial misalignment mechanism is subdominant compared to the thermal production for  $f_a = 10^{10}$  GeV. Note that the dependence of the abundance on  $f_a$  is different between the two production processes, and that for slightly larger values of  $f_a$ , the initial misalignment mechanism can dominate over the thermal production. This is the case if  $q_e$  is comparable to  $\sim 3$  or larger.

Lastly we consider the case of  $q_e = 0$ . In this case the preferred value of  $f_a$  is about  $10^5$  GeV, and the thermal production always dominates over the initial misalignment mechanism unless the anharmonic effect becomes significant [22, 23, 24, 25]. For  $T_R$  above the weak scale, the PNGBs are thermalized. For  $m_\mu < T < m_\tau$ , the PNGBs can be produced by scattering processes such as  $\mu + \gamma \rightarrow \mu + a$  with a rate given by

$$\Gamma_{\mu+\gamma \rightarrow \mu+a} \sim \alpha_{em} \frac{m_\mu^2}{f_a^2} T, \quad (13)$$

where  $T$  is the temperature. The production through the above process is most efficient at  $T = m_\mu$ , and the production rate exceeds the Hubble parameter at that time if

$$f_a \lesssim 4 \times 10^7 \text{ GeV}. \quad (14)$$

Therefore, for  $T_R \gtrsim m_\mu$ , the PNGBs are thermalized, and we need an additional entropy dilution by a factor of 40.<sup>5</sup> If  $T_R = \mathcal{O}(10)$  MeV, it is possible to produce the right amount of PNGBs to account for the observed dark matter abundance.

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<sup>4</sup> Recently, the BICEP2 experiment found the primordial B-mode polarization, implying that the inflation scale is about  $H_{\text{inf}} \sim 10^{14}$  GeV [21]. If this is true, the global  $U(1)_F$  symmetry must become spontaneously broken after inflation to avoid generating too large isocurvature perturbations. In this case, one needs to introduce extra breaking terms to avoid the cosmological catastrophe induced by domain walls.

<sup>5</sup> If the PNGB mass is of  $\mathcal{O}(0.1)$  eV or lighter, there is no problem even if it is thermalized. It would contribute to hot dark matter [26, 27] or the effective neutrino species  $\Delta N_{\text{eff}} \simeq 0.39$  [6]. Their existence are favored by recent observations [28, 29, 30, 31]. Interestingly, hot dark matter or dark radiation can relax the tension between BICEP2 and Planck.

	$e_R$	$\mu_R$	$\tau_R$	$\ell_1$	$\ell_2$	$\ell_3$
$Q$	$-a$	$-b$	$0$	$c$	$d$	$0$

Table 1: The charge assignment of leptons under the global  $U(1)_F$  flavor symmetry.

## 4 Anomaly-free flavor model for leptons

In this section we build anomaly-free flavor models for leptons. For simplicity we focus on a case in which electrons and muons are charged under the  $U(1)_F$  symmetry, while tau leptons are neutral. The extension to a more general charge assignment is straightforward.

Let us parametrize the global  $U(1)_F$  charges of  $e_i$  and  $\ell_j$  as  $Q(e_i) = (-a, -b, 0)$  and  $Q(\ell_j) = (c, d, 0)$ , where  $e_i$  and  $\ell_j$  are the right-handed charged-lepton singlet and the left-handed lepton doublet, respectively, and the subindices  $i, j = 1, 2, 3$  represent the generation. The charge assignment is also shown in Table 1. As long as there are no other fermions charged under both the global  $U(1)_F$  and SM gauge symmetries, the absence of the SM gauge anomalies requires

$$a + b = 0 \tag{15}$$

$$c + d = 0. \tag{16}$$

In order to write down Yukawa interactions for leptons, we need Higgs fields charged under the  $U(1)_F$  symmetry. Although not mandatory, let us seek the charge assignment, for which the off-diagonal elements are forbidden by the  $U(1)_F$  symmetry. We introduce three Higgs doublets,  $H(0)$ ,  $H(a+c)$ , and  $H(-a-c)$ , and require the following conditions:

$$a \neq 0 \tag{17}$$

$$c \neq 0 \tag{18}$$

$$a \neq c \tag{19}$$

$$2a + c \neq 0 \tag{20}$$

$$a + 2c \neq 0 \tag{21}$$

For any charged assignment satisfying the above conditions, the Yukawa interactions take the diagonal form,

$$\mathcal{L} \supset y_e \bar{e}_R \ell_1 H(-a-c) + y_\mu \bar{\mu}_R \ell_2 H(a+c) + y_\tau \bar{\tau}_R \ell_3 H(0) + \text{h.c.} \tag{22}$$



Let us normalize the global  $U(1)_F$  charge so that  $c = 1$ . Then the above conditions from (17) to (21) read  $a \neq 0, 1, -2$ , and so, the allowed integer values of  $a$  are  $a = -1, +2, \pm 3, \dots$ . Let us take up the first two cases, namely,  $a = -1$  and  $a = 2$ .

#### 4.1 Case of $a = -1$

In this case the global  $U(1)_F$  symmetry is identical to  $L_e - L_\mu$ , and there is only one Higgs doublet,  $H(0)$ . Then the PNGB does not have direct couplings with charged leptons like Eq. (1), as it does not reside in the phase of  $H(0)$ .

Let us extend the SM to include right-handed neutrinos  $N_i$ . If we assign the global  $U(1)_F$  charges as  $Q(N_i) = (1, -1, 0)$ , the neutrino Yukawa interaction is diagonal;

$$\mathcal{L} \supset y_i^N \bar{N}_i \ell_i \tilde{H}(0) + \text{h.c.} \quad (23)$$

where  $\tilde{H}(0) = i\sigma_2 H(0)^*$ . The observed large neutrino mixings can be explained if the Majorana mass matrix for  $N_i$  contains large off-diagonal elements. To this end we introduce  $U(1)_{B-L}$  gauge symmetry and the  $B-L$  Higgs fields,  $\phi(0)$ ,  $\phi(\pm 1)$ , and  $\phi(\pm 2)$ , where the numbers in the parentheses represent the  $U(1)_F$  charge and they are assumed to have a common  $B-L$  charge  $+2$ . If these  $B-L$  Higgs fields develop non-zero vacuum expectation values (VEVs), the  $U(1)_F$  symmetry is spontaneously broken, and the Majorana mass matrix for  $N_i$  is induced as

$$-\mathcal{L} \supset \frac{1}{2} (M_N)_{ij} \bar{N}_i^c N_j + \text{h.c.} \quad (24)$$

with

$$M_N \sim \begin{pmatrix} \phi(-2) & \phi(0) & \phi(-1) \\ \phi(0) & \phi(2) & \phi(1) \\ \phi(-1) & \phi(1) & \phi(0) \end{pmatrix}, \quad (25)$$

where the  $B-L$  Higgs fields are understood to represent their VEVs, and we have dropped  $O(1)$  numerical coefficient in each element. If the VEVs are comparable to each other, the large neutrino mixing angles are realized. The light neutrino masses can be explained by the seesaw mechanism [32].

The PNGB resides in the phase of  $\phi(1)$  and  $\phi(2)$ , and the decay constant  $f_a$  is approximately given by their VEVs. In fact, the PNGB in this case is similar to the majoron. The cosmological constraints on the majoron dark matter were studied in e.g. Ref. [33].

One can also introduce the Higgs portal couplings  $\sim |\phi|^2 |H|^2$ . The situation would be similar to the model proposed by Weinberg [6]. For a certain set of parameters, massless PNGBs would contribute the effective neutrino species,  $\Delta N_{\text{eff}}$ .

## 4.2 Case of $a = +2$

In this case the charge assignment is  $Q(e_i) = (-2, 2, 0)$  and  $Q(\ell_i) = (1, -1, 0)$ , and there are three Higgs doublets,  $H(0)$  and  $H(\pm 3)$ . The charged lepton Yukawa interactions are given by

$$\mathcal{L} \supset y_e \bar{e}_R \ell_1 H(-3) + y_2 \bar{\mu}_R \ell_2 H(3) + y_3 \bar{\tau}_R \ell_3 H(0) + \text{h.c.} \quad (26)$$

The previous argument on the neutrino Yukawa interaction and the right-handed neutrino mass matrix can be applied to the present case, and the observed large neutrino mixing as well as the neutrino mass scale can be similarly explained.

The global  $U(1)_F$  symmetry is spontaneously broken by both the Higgs doublets and the  $B - L$  Higgs fields. We assume that the symmetry breaking scale is of order  $10^{10}$  GeV (or smaller). Then, while the PNGB resides mainly in the phase of  $\phi(1)$  and  $\phi(2)$ , it also appears in the phase of  $H(3)$  and  $H(-3)$ , and so, electrons and muons are coupled to the PNGB in the low energy as in Eq. (1). Since the PNGB does not have (sizable) couplings with gluons, photons, and quarks, the astrophysical bounds are considered to be rather weak.

A couple of comments are in order. In order to give a mass to the PNGB, one needs an explicit  $U(1)_F$  symmetry breaking. It is interesting to note that the following term

$$\mathcal{L}_{\text{breaking}} = m^2 H(-3)^\dagger H(3) + \text{h.c.} \quad (27)$$

breaks the  $U(1)_F$  symmetry down to the subgroup  $Z_6$ , giving rise to a PNGB mass  $m_a \sim \mathcal{O}(1)$  keV for  $m \sim \langle H(-3) \rangle \sim \langle H(3) \rangle \sim 10^2$  GeV and  $f_a = \mathcal{O}(10^{10})$  GeV. Therefore, the PNGB associated with anomaly-free flavor symmetry broken at  $f_a = \mathcal{O}(10^{10})$  GeV nicely explains both the mass and the lifetime suggested by the observed 3.5 keV X-ray line.

The off-diagonal elements of the charged lepton Yukawa matrix receive non-zero contributions, as the  $U(1)_F$  symmetry is spontaneously broken. Their contributions to the lepton-flavor violating processes, however, are negligible in our model.

In the presence of the  $B - L$  Higgs fields, there are in general mixings between  $H$  and  $\phi$ . Such mixings are assumed to be small in our context to keep the hierarchy between the weak scale and the flavor symmetry breaking scale. Also we assume that the lightest Higgs has a property similar to the SM Higgs and the other Higgs fields are so heavy that they evade the current collider search. Some of them, however, may be within the reach of LHC and/or ILC.

## 5 Discussion and conclusions

Some comments and discussions are in order. In the case of  $q_e \neq 0$ ,  $f_a = \mathcal{O}(10^{9-10})$  GeV is needed to explain the 3.5 keV X-ray line. Since the couplings to photons, gluons and nucleons are suppressed, the PNGBs avoid various astrophysical and ground-based constraints. Still, it may be possible to find them in the future. Interestingly, there is a hint for an extra cooling of white dwarfs, which can be explained by light PNGBs coupled to electrons with the decay constant in this range [34].<sup>6</sup> If such light PNGBs are coupled with electrons but not with photons, it is possible that they are copiously produced in the Sun, but cannot be detected by experiments using the magnetic field like the CAST experiment [35].

In the case of  $q_e = 0$  and  $q_\mu \neq 0$ , the preferred value of  $f_a$  is of order  $10^5$  GeV, much smaller than the previous case. Still, as the effective PNGB coupling to the photon is so weak that the constraint from the cooling of horizontal branch stars can be satisfied [36]. On the other hand, the bound from supernova cooling will be more non-trivial since the PNGB couples to muons directly and the muons might be abundant in the supernova core [37, 38]. Although the muon abundance depends sensitively on the temperature, the preferred value of  $f_a$  may be in tension with the observation. As a rough estimate, we refer to the constraint on the majoron coupling constants to neutrinos from the supernova cooling: it is bounded as  $g_{ee} \lesssim 10^{-6}$  where  $g_{ee}$  is the yukawa coupling between the majoron and electron neutrinos [39]. In our case, the effective coupling constant between the PNGB and the muon reads  $m_\mu/f_a \sim 10^{-6}$ . A more detailed study is needed to test the viability

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<sup>6</sup> In Ref. [34], the QCD axion was considered, and so, the cooling rate due to the 7 keV axion can be much smaller for the same decay rate. There may be another PNGB, if the flavor symmetry group larger than  $U(1)_F$ .

of this model.

So far we have considered the  $U(1)_F$  flavor symmetry, under which only leptons are charged, and we constructed models in which the lepton mass matrix is (almost) diagonal. It is possible to extend the models to allow larger off-diagonal terms, or to extend the flavor symmetry to the quark sector, by enlarging the flavor symmetry and adding more Higgs fields. If the actual flavor symmetry group is larger than  $U(1)_F$  and if it is broken at a scale of  $\mathcal{O}(10^9\text{--}10^6)$  GeV, there may be more PNGBs with different masses with or without couplings to photons and/or gluons. Then it may be possible to provide a unified picture of the QCD axion well as other PNGBs. In this case the light PNGBs can be searched for by flavor-changing processes such as  $\tau \rightarrow \mu + a$ ,  $\mu^+ \rightarrow e^+ + a$ ,  $K^+ \rightarrow \pi^+ + a$  [40].

We have pursued a possibility that a PNGB is lurking below the intermediate scale, evading the astrophysical bounds. Along this line we have proposed flavor models based on an anomaly-free  $U(1)_F$  symmetry, where the PNGB is preferentially coupled to the leptons. In particular, its anomalous couplings to gluons and photons are absent, greatly relaxing the astrophysical bounds. We have also pointed out that, although suppressed, the PNGB coupling to photons is induced by threshold corrections. Interestingly, the recent hint for the X-ray line at about 3.5 keV [8, 9] can be explained by the PNGB dark matter with  $m_a \simeq 7$  keV for the decay constant  $f_a = 10^9\text{--}10^6$  GeV ( $f_a = 10^{5\text{--}6}$  GeV) if electrons are (not) charged under the flavor symmetry.

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